Lecture 17

Dielectric Waveguides

Before we embark on the study of dielectric waveguides, we will revisit the transverse resonance again. The transverse resonance condition allows one to derive the guidance conditions for a dielectric waveguide easily without having to match the boundary conditions at the interface again: The boundary conditions are already used when deriving the Fresnel reflection coefficients, and hence they are embedded in these reflection coefficients. Much of the materials in this lecture can be found in [31, 38, 75].

17.1 Generalized Transverse Resonance Condition

The guidance conditions, the transverse resonance condition given previously, can also be derived for the more general case. The generalized transverse resonance condition is a powerful condition that can be used to derive the guidance condition of a mode in a layered medium.

To derive this condition, we first have to realize that a guided mode in a waveguide is due to the coherent or constructive interference of the waves. This implies that if a plane wave starts at position 1 (see Figure 17.1)[1] and is multiply reflected as shown, it will regain its original phase in the x direction at position 5. Since this mode progresses in the z direction, all these waves (also known as partial waves) are in phase in the z direction by the phase matching condition. Otherwise, the boundary conditions cannot be satisfied. That is, waves at 1 and 5 will gain the same phase in the z direction. But, for them to add coherently or interfere coherently in the x direction, the transverse phase at 5 must be the same as 1.

Assuming that the wave starts with amplitude 1 at position 1, it will gain a transverse phase of $e^{-j\beta_{0x}t}$ when it reaches position 2. Upon reflection at $x = x_2$, at position 3, the wave becomes $\tilde{R}_+ e^{-j\beta_{0x}t}$ where $\tilde{R}_+$ is the generalized reflection coefficient at the right interface of Region 0. Finally, at position 5, it becomes $\tilde{R}_- \tilde{R}_+ e^{-2j\beta_{0x}t}$ where $\tilde{R}_-$ is the generalized reflection coefficient at the left interface of Region 0. For constructive interference to occur

[1]The waveguide convention is to assume the direction of propagation to be z. Since we are analyzing a guided mode in a layered medium, z axis is as shown in this figure.

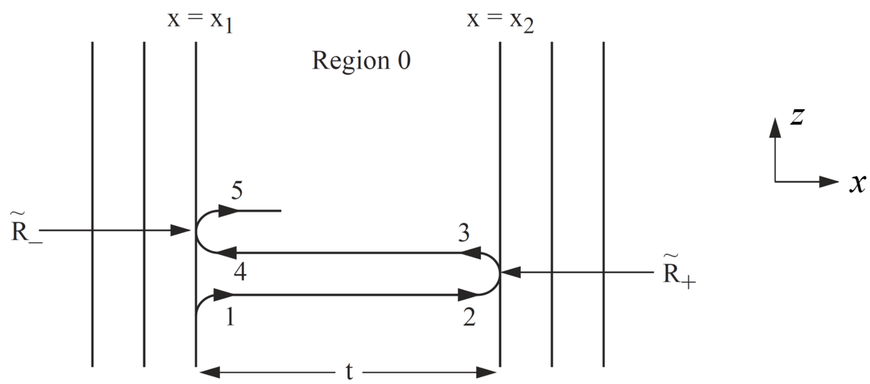

Figure 17.1: The transverse resonance condition for a layered medium. The phase of the wave at position 5 should be equal to the transverse phase at position 1.

or for the mode to exist, we require that

$$\tilde{R}_{-}\tilde{R}_{+}e^{-2j\beta_{0x}t} = 1 \tag{17.1.1}$$

The above is the generalized transverse resonance condition for the guidance condition for a plane wave mode traveling in a layered medium.

In (17.1.1), a metallic wall has a reflection coefficient of 1 for a TM wave, hence if $\tilde{R}_{+}$ is 1, Equation (17.1.1) becomes

$$1 - \tilde{R}_{-}e^{2-j\beta_{0x}t} = 0. \tag{17.1.2}$$

On the other hand, in (17.1.1), a metallic wall has a reflection coefficient of -1, for TE wave, and Equation (17.1.1) becomes

$$1 + \tilde{R}_{-}e^{2-j\beta_{0x}t} = 0. \tag{17.1.3}$$

17.2 Dielectric Waveguide

The most important dielectric waveguide of the modern world is the optical fiber, whose invention was credited to Charles Kao [91]. He was awarded the Nobel prize in 2009 [102]. However, the analysis of the optical fiber requires analysis in cylindrical coordinates and the use of special functions such as Bessel functions. In order to capture the essence of dielectric waveguides, one can study the slab dielectric waveguide, which shares many salient features with the optical fiber. This waveguide is also used as thin-film optical waveguides (see Figure 17.2). We start with analyzing the TE modes in this waveguide.

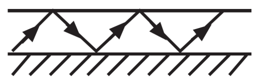

Figure 17.2: An optical thin-film waveguide is made by coating a thin dielectric film or sheet on a metallic surface. The wave is guided by total internal reflection at the top interface, and by metallic reflection at the bottom interface.

17.2.1 TE Case

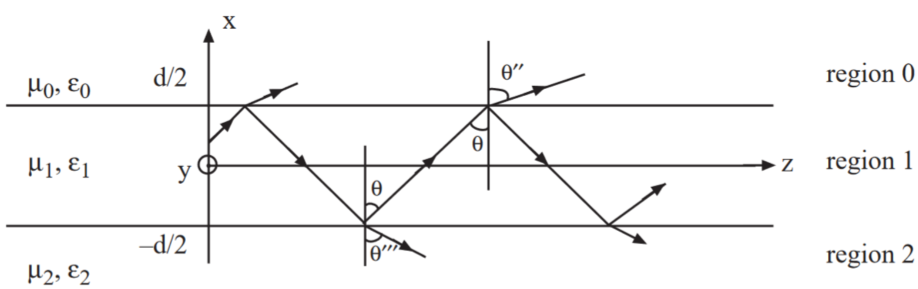

Figure 17.3: Schematic for the analysis of a guided mode in the dielectric waveguide. Total internal reflection occurs at the top and bottom interfaces. If the waves add coherently, the wave is guided along the dielectric slab.

We shall look at the application of the transverse resonance condition to a TE wave guided in a dielectric waveguide. Again, we assume the direction of propagation of the guided mode to be in the z direciton in accordance to convention. Specializing the above equation to the dielectric waveguide shown in Figure 17.3, we have the guidance condition as

$$1 = R_{10} R_{12} e^{-2j\beta_{1x} d} \tag{17.2.1}$$

where d is the thickness of the dielectric slab. Guidance of a mode is due to total internal reflection, and hence, we expect Region 1 to be optically more dense (in terms of optical refractive indices)[2] than region 0 and 2.

To simplify the analysis further, we assume Region 2 to be the same as Region 0. The new guidance condition is then

$$1 = R_{10}^2 e^{-2j\beta_{1x} d} \tag{17.2.2}$$

[2]Optically more dense means higher optical refractive index, or higher dielectric constant.

Also, we assume that $\varepsilon_1 > \varepsilon_0$ so that total internal reflection occurs at both interfaces as the wave bounces around so that $\beta_{0x} = -j\alpha_{0x}$. Therefore, for TE polarization, the single-interface reflection coefficient is

$$R_{10} = \frac{\mu_0\beta_{1x} - \mu_1\beta_{0x}}{\mu_0\beta_{1x} + \mu_1\beta_{0x}} = \frac{\mu_0\beta_{1x} + j\mu_1\alpha_{0x}}{\mu_0\beta_{1x} - j\mu_1\alpha_{0x}} = e^{j\theta_{TE}} \tag{17.2.3}$$

where θ_{TE} is the Goos-Hanschen shift for total internal reflection. It is given by

$$\theta_{TE} = 2\tan^{-1}\left(\frac{\mu_1\alpha_{0x}}{\mu_0\beta_{1x}}\right) \tag{17.2.4}$$

The guidance condition for constructive interference according to (17.2.1) is such that

$$2\theta_{TE} - 2\beta_{1x}d = 2n\pi \tag{17.2.5}$$

From the above, dividing it by four, and taking its tangent, we get

$$\tan\left(\frac{\theta_{TE}}{2}\right) = \tan\left(\frac{n\pi}{2} + \frac{\beta_{1x}d}{2}\right) \tag{17.2.6}$$

or

$$\frac{\mu_1\alpha_{0x}}{\mu_0\beta_{1x}} = \tan\left(\frac{n\pi}{2} + \frac{\beta_{1x}d}{2}\right) \tag{17.2.7}$$

The above gives rise to

$$\mu_1\alpha_{0x} = \mu_0\beta_{1x}\tan\left(\frac{\beta_{1x}d}{2}\right), \qquad n \text{ even} \tag{17.2.8}$$

$$-\mu_1\alpha_{0x} = \mu_0\beta_{1x}\cot\left(\frac{\beta_{1x}d}{2}\right), \qquad n \text{ odd} \tag{17.2.9}$$

It can be shown that when n is even, the mode profile is even, whereas when n is odd, the mode profile is odd. The above can also be rewritten as

$$\frac{\mu_0}{\mu_1}\frac{\beta_{1x}d}{2}\tan\left(\frac{\beta_{1x}d}{2}\right) = \frac{\alpha_{0x}d}{2}, \qquad \text{even modes} \tag{17.2.10}$$

$$-\frac{\mu_0}{\mu_1}\frac{\beta_{1x}d}{2}\cot\left(\frac{\beta_{1x}d}{2}\right) = \frac{\alpha_{0x}d}{2}, \qquad \text{odd modes} \tag{17.2.11}$$

Using the fact that $-\alpha_{0x}^2 = \beta_0^2 - \beta_z^2$, and that $\beta_{1x}^2 = \beta_1^2 - \beta_z^2$, eliminating β_z from these two equations, one can show that

$$\alpha_{0x} = [\omega^2(\mu_1\epsilon_1 - \mu_0\epsilon_0) - \beta_{1x}^2]^{\frac{1}{2}} \tag{17.2.12}$$

and (17.2.10) and (17.2.11) become

$$\begin{aligned}\frac{\mu_0}{\mu_1}\frac{\beta_{1x}d}{2}\tan\left(\frac{\beta_{1x}d}{2}\right) &= \frac{\alpha_{0x}d}{2}\\ &= \sqrt{\omega^2(\mu_1\epsilon_1-\mu_0\epsilon_0)\frac{d^2}{4}-\left(\frac{\beta_{1x}d}{2}\right)^2}, \quad \text{even modes} \qquad (17.2.13)\\ -\frac{\mu_0}{\mu_1}\frac{\beta_{1x}d}{2}\cot\left(\frac{\beta_{1x}d}{2}\right) &= \frac{\alpha_{0x}d}{2}\\ &= \sqrt{\omega^2(\mu_1\epsilon_1-\mu_0\epsilon_0)\frac{d^2}{4}-\left(\frac{\beta_{1x}d}{2}\right)^2}, \quad \text{odd modes} \qquad (17.2.14)\end{aligned}$$

We can solve the above graphically by plotting

$$y_1 = \frac{\mu_0}{\mu_1}\frac{\beta_{1x}d}{2}\tan\left(\frac{\beta_{1x}d}{2}\right) \quad \text{even modes} \qquad (17.2.15)$$

$$y_2 = -\frac{\mu_0}{\mu_1}\frac{\beta_{1x}d}{2}\cot\left(\beta_{1x}\frac{d}{2}\right) \quad \text{odd modes} \qquad (17.2.16)$$

$$y_3 = \left[\omega^2(\mu_1\epsilon_1-\mu_0\epsilon_0)\frac{d^2}{4}-\left(\frac{\beta_{1x}d}{2}\right)^2\right]^{\frac{1}{2}} = \frac{\alpha_{0x}d}{2} \qquad (17.2.17)$$

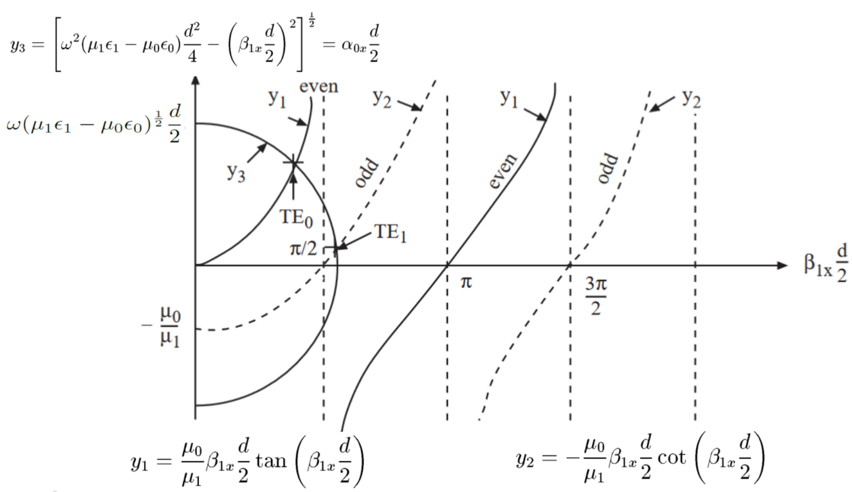

Figure 17.4: A way to solve (17.2.13) and (17.2.13) is via a graphical method. In this method, both the right-hand side and the left-hand side of the equations are plotted on the same plot. The solutions are the points of intersection of these plots.

In the above, y_3 is the equation of a circle; the radius of the circle is given by

$$\omega(\mu_1\epsilon_1 - \mu_0\epsilon_0)^{\frac{1}{2}}\frac{d}{2}. \tag{17.2.18}$$

The solutions to (17.2.13) and (17.2.14) are given by the intersections of y_3 with y_1 and y_2. We note from (17.2.1) that the radius of the circle can be increased in three ways: (i) by increasing the frequency, (ii) by increasing the contrast $\frac{\mu_1\epsilon_1}{\mu_0\epsilon_0}$, and (iii) by increasing the thickness d of the slab.[3] The mode profiles of the first two modes are shown in Figure 17.5.

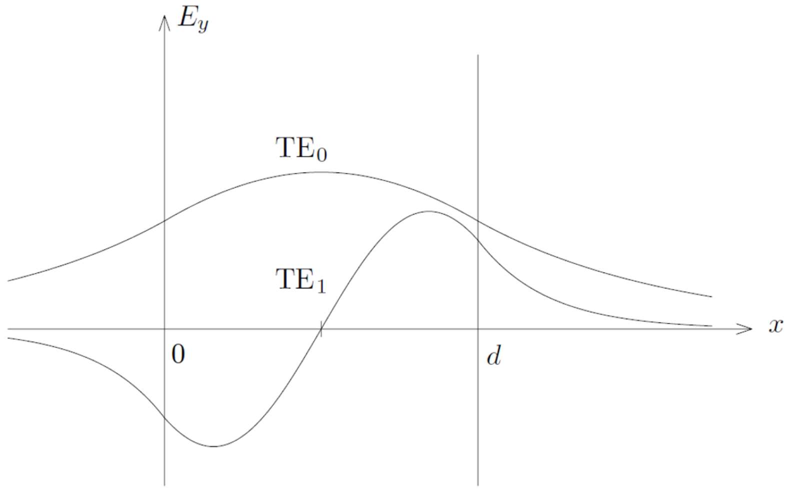

Figure 17.5: Mode profiles of the TE$_0$ and TE$_1$ modes of a dielectric slab waveguide (courtesy of J.A. Kong [31]).

When $\beta_{0x} = -j\alpha_{0x}$, the reflection coefficient for total internal reflection is

$$R_{10}^{TE} = \frac{\mu_0\beta_{1x} + j\mu_1\alpha_{0x}}{\mu_0\beta_{1x} - j\mu_1\alpha_{0x}} = \exp\left[+2j\tan^{-1}\left(\frac{\mu_1\alpha_{0x}}{\mu_0\beta_{1x}}\right)\right] \tag{17.2.19}$$

and $\left|R_{10}^{TE}\right| = 1$. Hence, the wave is guided by total internal reflections.

Cut-off occurs when the total internal reflection ceases to occur, i.e. when the frequency decreases such that $\alpha_{0x} = 0$.

From Figure 17.4, we see that $\alpha_{0x} = 0$ when

$$\omega(\mu_1\epsilon_1 - \mu_0\epsilon_0)^{\frac{1}{2}}\frac{d}{2} = \frac{m\pi}{2}, \qquad m = 0, 1, 2, 3, \ldots \tag{17.2.20}$$

[3]These features are also shared by the optical fiber.

or

$$\omega_{mc} = \frac{m\pi}{d(\mu_1\epsilon_1 - \mu_0\epsilon_0)^{\frac{1}{2}}}, \qquad m = 0, 1, 2, 3, \ldots \tag{17.2.21}$$

The mode that corresponds to the m-th cut-off frequency above is labeled the TE_m mode. Thus TE_0 mode is the mode that has no cut-off or propagates at all frequencies. This is shown in Figure 17.6 where the TE mode profiles are similar since they are dual to each other. The boundary conditions at the dielectric interface is that the field and its normal derivative have to be continuous. The TE_0 or TM_0 mode can satisfy this boundary condition at all frequencies, but not the TE_1 or TM_1 mode. At the cut-off frequency, the field outside the slab has to become flat implying the $\alpha_{0x} = 0$ implying no guidance.

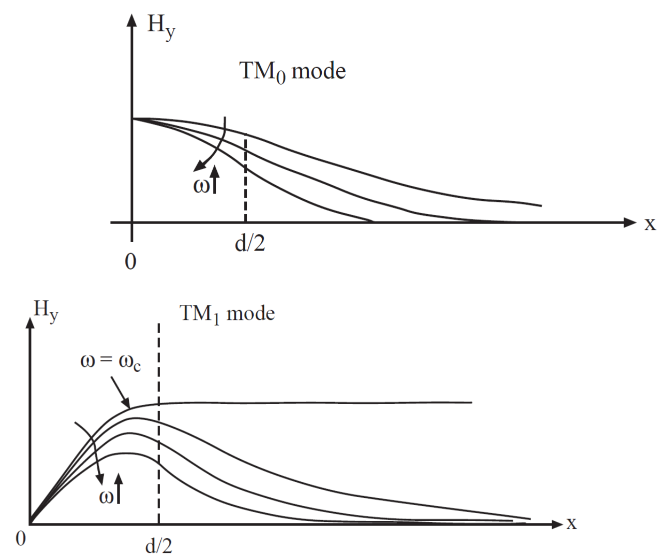

Figure 17.6: The TE modes are dual to the TM modes and have similar mode profiles.

At cut-off, $\alpha_{0x} = 0$, and from the dispersion relation that $\alpha_{0x}^2 = \beta_z^2 - \beta_0^2$,

$$\beta_z = \omega\sqrt{\mu_0\epsilon_0},$$

for all the modes. Hence, both the group and the phase velocities are that of the outer region. This is because when $\alpha_{0x} = 0$, the wave is not evanescent outside, and most of the energy of the mode is carried by the exterior field.

When $\omega \to \infty$, the radius of the circle in the plot of y_3 becomes increasingly larger. As seen from Figure 17.4, the solution for $\beta_{1x} \to \frac{n\pi}{d}$ for all the modes. From the dispersion relation for Region 1,

$$\beta_z = \sqrt{\omega^2\mu_1\epsilon_1 - \beta_{1x}^2} \approx \omega\sqrt{\mu_1\epsilon_1}, \qquad \omega \to \infty \tag{17.2.22}$$

Hence the group and phase velocities approach that of the dielectric slab. This is because when $\omega \to \infty$, $\alpha_{0x} \to \infty$, implying that the fields are trapped or confined in the slab and propagating within it. Because of this, the dispersion diagram of the different modes appear as shown in Figure 17.7. In this figure,[4] k_{c1}, k_{c2}, and k_{c3} are the cut-off wave number or frequency of the first three modes. Close to cut-off, the field is traveling mostly outside the waveguide, and $k_z \approx \omega\sqrt{\mu_0\varepsilon_0}$, and both the phase and group velocities approach that of the outer medium as shown in the figure. When the frequency increases, the mode is tightly confined in the dielectric slab, and $k_z \approx \omega\sqrt{\mu_1\varepsilon_1}$. Both the phase and group velocities approach that of Region 1 as shown.

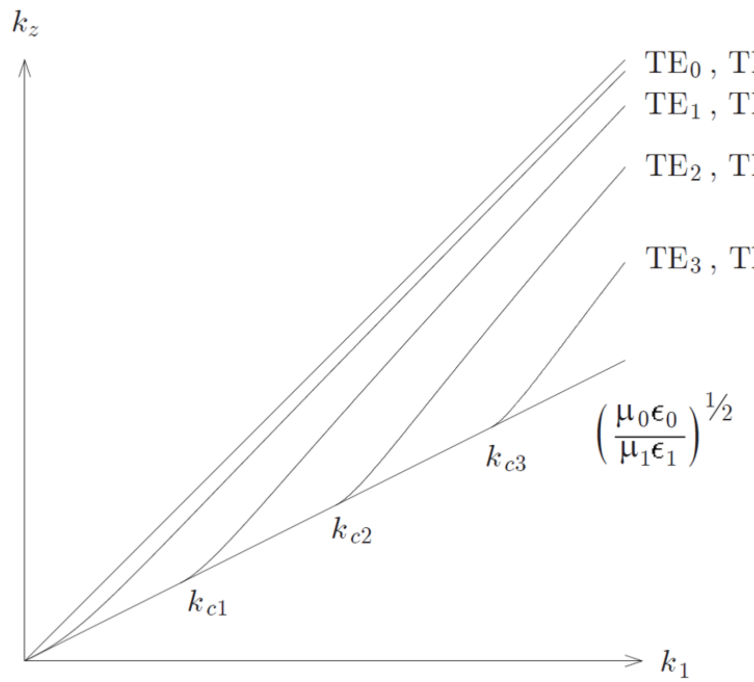

Figure 17.7: Here, we have k_z versus k_1 plot for dielectric slab waveguide. Near its cut-off, the energy of the mode is in the outer region, and hence, its group velocity is close to that of the outer region. At high frequencies, the mode is tightly bound to the slab, and its group velocity approaches that of the dielectric slab (courtesy of J.A. Kong [31]).

17.2.2 TM Case

For the TM case, a similar guidance condition analogous to (17.2.1) can be derived but with the understanding that the reflection coefficients in (17.2.1) are now TM reflection coefficients.

[4] Please note again that in this course, we will use β and k interchangeably for wavenumbers.

Similar derivations show that the above guidance condition, for $\epsilon_2 = \epsilon_0$, $\mu_2 = \mu_0$, reduces to

$$\frac{\epsilon_0}{\epsilon_1}\beta_{1x}\frac{d}{2}\tan\beta_{1x}\frac{d}{2} = \sqrt{\omega^2(\mu_1\epsilon_1 - \mu_0\epsilon_0)\frac{d^2}{4} - \left(\beta_{1x}\frac{d}{2}\right)^2}, \qquad \text{even modes} \tag{17.2.23}$$

$$-\frac{\epsilon_0}{\epsilon_1}\beta_{1x}\frac{d}{2}\cot\beta_{1x}\frac{d}{2} = \sqrt{\omega^2(\mu_1\epsilon_1 - \mu_0\epsilon_0)\frac{d^2}{4} - \left(\beta_{1x}\frac{d}{2}\right)^2}, \qquad \text{odd modes} \tag{17.2.24}$$

Note that for equation (17.2.1), when we have two parallel metallic plates, $R^{TM} = 1$, and $R^{TE} = -1$, and the guidance condition becomes

$$1 = e^{-2j\beta_{1x}d} \quad \Rightarrow \beta_{1x} = \frac{m\pi}{d}, \qquad m = 0, 1, 2, \ldots, \tag{17.2.25}$$

17.2.3 A Note on Cut-Off of Dielectric Waveguides

The concept of cut-off in dielectric waveguides is quite different from that of hollow waveguides that we shall learn next. A mode is guided in a dielectric waveguide if the wave is trapped inside, in this case, the dielectric slab. The trapping is due to the total internal reflections at the top and the bottom interface of the waveguide. WShen total internal reflection ceases to occur at any of the two interfaces, the wave is not guided or trapped inside the dielectric slab anymore. This happens when $\alpha_{ix} = 0$ where i can indicate the top-most or the bottom-most region. In other words, the wave ceases to be evanescent in Region i.

Bibliography

[1] J. A. Kong, *Theory of electromagnetic waves.* New York, Wiley-Interscience, 1975.

[2] A. Einstein *et al.*, "On the electrodynamics of moving bodies," *Annalen der Physik*, vol. 17, no. 891, p. 50, 1905.

[3] P. A. M. Dirac, "The quantum theory of the emission and absorption of radiation," *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, vol. 114, no. 767, pp. 243–265, 1927.

[4] R. J. Glauber, "Coherent and incoherent states of the radiation field," *Physical Review*, vol. 131, no. 6, p. 2766, 1963.

[5] C.-N. Yang and R. L. Mills, "Conservation of isotopic spin and isotopic gauge invariance," *Physical review*, vol. 96, no. 1, p. 191, 1954.

[6] G. t'Hooft, *50 years of Yang-Mills theory.* World Scientific, 2005.

[7] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation.* Princeton University Press, 2017.

[8] F. Teixeira and W. C. Chew, "Differential forms, metrics, and the reflectionless absorption of electromagnetic waves," *Journal of Electromagnetic Waves and Applications*, vol. 13, no. 5, pp. 665–686, 1999.

[9] W. C. Chew, E. Michielssen, J.-M. Jin, and J. Song, *Fast and efficient algorithms in computational electromagnetics.* Artech House, Inc., 2001.

[10] A. Volta, "On the electricity excited by the mere contact of conducting substances of different kinds. in a letter from Mr. Alexander Volta, FRS Professor of Natural Philosophy in the University of Pavia, to the Rt. Hon. Sir Joseph Banks, Bart. KBPR S," *Philosophical transactions of the Royal Society of London*, no. 90, pp. 403–431, 1800.

[11] A.-M. Ampère, *Exposé méthodique des phénomènes électro-dynamiques, et des lois de ces phénomènes.* Bachelier, 1823.

[12] ——, *Mémoire sur la théorie mathématique des phénomènes électro-dynamiques uniquement déduite de l'expérience: dans lequel se trouvent réunis les Mémoires que M. Ampère a communiqués à l'Académie royale des Sciences, dans les séances des 4 et*

26 décembre 1820, 10 juin 1822, 22 décembre 1823, 12 septembre et 21 novembre 1825. Bachelier, 1825.

[13] B. Jones and M. Faraday, *The life and letters of Faraday.* Cambridge University Press, 2010, vol. 2.

[14] G. Kirchhoff, "Ueber die auflösung der gleichungen, auf welche man bei der untersuchung der linearen vertheilung galvanischer ströme geführt wird," *Annalen der Physik*, vol. 148, no. 12, pp. 497–508, 1847.

[15] L. Weinberg, "Kirchhoff's' third and fourth laws'," *IRE Transactions on Circuit Theory*, vol. 5, no. 1, pp. 8–30, 1958.

[16] T. Standage, *The Victorian Internet: The remarkable story of the telegraph and the nineteenth century's online pioneers.* Phoenix, 1998.

[17] J. C. Maxwell, "A dynamical theory of the electromagnetic field," *Philosophical transactions of the Royal Society of London*, no. 155, pp. 459–512, 1865.

[18] H. Hertz, "On the finite velocity of propagation of electromagnetic actions," *Electric Waves*, vol. 110, 1888.

[19] M. Romer and I. B. Cohen, "Roemer and the first determination of the velocity of light (1676)," *Isis*, vol. 31, no. 2, pp. 327–379, 1940.

[20] A. Arons and M. Peppard, "Einstein's proposal of the photon concept–a translation of the Annalen der Physik paper of 1905," *American Journal of Physics*, vol. 33, no. 5, pp. 367–374, 1965.

[21] A. Pais, "Einstein and the quantum theory," *Reviews of Modern Physics*, vol. 51, no. 4, p. 863, 1979.

[22] M. Planck, "On the law of distribution of energy in the normal spectrum," *Annalen der physik*, vol. 4, no. 553, p. 1, 1901.

[23] Z. Peng, S. De Graaf, J. Tsai, and O. Astafiev, "Tuneable on-demand single-photon source in the microwave range," *Nature communications*, vol. 7, p. 12588, 2016.

[24] B. D. Gates, Q. Xu, M. Stewart, D. Ryan, C. G. Willson, and G. M. Whitesides, "New approaches to nanofabrication: molding, printing, and other techniques," *Chemical reviews*, vol. 105, no. 4, pp. 1171–1196, 2005.

[25] J. S. Bell, "The debate on the significance of his contributions to the foundations of quantum mechanics, Bells Theorem and the Foundations of Modern Physics (A. van der Merwe, F. Selleri, and G. Tarozzi, eds.)," 1992.

[26] D. J. Griffiths and D. F. Schroeter, *Introduction to quantum mechanics.* Cambridge University Press, 2018.

[27] C. Pickover, *Archimedes to Hawking: Laws of science and the great minds behind them.* Oxford University Press, 2008.

[28] R. Resnick, J. Walker, and D. Halliday, *Fundamentals of physics.* John Wiley, 1988.

[29] S. Ramo, J. R. Whinnery, and T. Duzer van, *Fields and waves in communication electronics, Third Edition.* John Wiley & Sons, Inc., 1995.

[30] J. L. De Lagrange, "Recherches d'arithmétique," *Nouveaux Mémoires de l'Académie de Berlin*, 1773.

[31] J. A. Kong, *Electromagnetic Wave Theory.* EMW Publishing, 2008.

[32] H. M. Schey, *Div, grad, curl, and all that: an informal text on vector calculus.* WW Norton New York, 2005.

[33] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman lectures on physics, Vols. I, II, & III: The new millennium edition.* Basic books, 2011, vol. 1,2,3.

[34] W. C. Chew, *Waves and fields in inhomogeneous media.* IEEE press, 1995.

[35] V. J. Katz, "The history of Stokes' theorem," *Mathematics Magazine*, vol. 52, no. 3, pp. 146–156, 1979.

[36] W. K. Panofsky and M. Phillips, *Classical electricity and magnetism.* Courier Corporation, 2005.

[37] T. Lancaster and S. J. Blundell, *Quantum field theory for the gifted amateur.* OUP Oxford, 2014.

[38] W. C. Chew, "Fields and waves: Lecture notes for ECE 350 at UIUC," *https://engineering.purdue.edu/wcchew/ece350.html*, 1990.

[39] C. M. Bender and S. A. Orszag, *Advanced mathematical methods for scientists and engineers I: Asymptotic methods and perturbation theory.* Springer Science & Business Media, 2013.

[40] J. M. Crowley, *Fundamentals of applied electrostatics.* Krieger Publishing Company, 1986.

[41] C. Balanis, *Advanced Engineering Electromagnetics.* Hoboken, NJ, USA: Wiley, 2012.

[42] J. D. Jackson, *Classical electrodynamics.* John Wiley & Sons, 1999.

[43] R. Courant and D. Hilbert, *Methods of Mathematical Physics: Partial Differential Equations.* John Wiley & Sons, 2008.

[44] L. Esaki and R. Tsu, "Superlattice and negative differential conductivity in semiconductors," *IBM Journal of Research and Development*, vol. 14, no. 1, pp. 61–65, 1970.

[45] E. Kudeki and D. C. Munson, *Analog Signals and Systems.* Upper Saddle River, NJ, USA: Pearson Prentice Hall, 2009.

[46] A. V. Oppenheim and R. W. Schafer, *Discrete-time signal processing.* Pearson Education, 2014.

[47] R. F. Harrington, *Time-harmonic electromagnetic fields.* McGraw-Hill, 1961.

[48] E. C. Jordan and K. G. Balmain, *Electromagnetic waves and radiating systems.* Prentice-Hall, 1968.

[49] G. Agarwal, D. Pattanayak, and E. Wolf, "Electromagnetic fields in spatially dispersive media," *Physical Review B*, vol. 10, no. 4, p. 1447, 1974.

[50] S. L. Chuang, *Physics of photonic devices.* John Wiley & Sons, 2012, vol. 80.

[51] B. E. Saleh and M. C. Teich, *Fundamentals of photonics.* John Wiley & Sons, 2019.

[52] M. Born and E. Wolf, *Principles of optics: electromagnetic theory of propagation, interference and diffraction of light.* Elsevier, 2013.

[53] R. W. Boyd, *Nonlinear optics.* Elsevier, 2003.

[54] Y.-R. Shen, *The principles of nonlinear optics.* New York, Wiley-Interscience, 1984.

[55] N. Bloembergen, *Nonlinear optics.* World Scientific, 1996.

[56] P. C. Krause, O. Wasynczuk, and S. D. Sudhoff, *Analysis of electric machinery.* McGraw-Hill New York, 1986.

[57] A. E. Fitzgerald, C. Kingsley, S. D. Umans, and B. James, *Electric machinery.* McGraw-Hill New York, 2003, vol. 5.

[58] M. A. Brown and R. C. Semelka, *MRI.: Basic Principles and Applications.* John Wiley & Sons, 2011.

[59] C. A. Balanis, *Advanced engineering electromagnetics.* John Wiley & Sons, 1999.

[60] Wikipedia, "Lorentz force," https://en.wikipedia.org/wiki/Lorentz_force/, accessed: 2019-09-06.

[61] R. O. Dendy, *Plasma physics: an introductory course.* Cambridge University Press, 1995.

[62] P. Sen and W. C. Chew, "The frequency dependent dielectric and conductivity response of sedimentary rocks," *Journal of microwave power*, vol. 18, no. 1, pp. 95–105, 1983.

[63] D. A. Miller, *Quantum Mechanics for Scientists and Engineers.* Cambridge, UK: Cambridge University Press, 2008.

[64] W. C. Chew, "Quantum mechanics made simple: Lecture notes for ECE 487 at UIUC," *http://wcchew.ece.illinois.edu/chew/course/QMAll20161206.pdf*, 2016.

[65] B. G. Streetman and S. Banerjee, *Solid state electronic devices.* Prentice hall Englewood Cliffs, NJ, 1995.

[66] Smithsonian, "This 1600-year-old goblet shows that the romans were nanotechnology pioneers," https://www.smithsonianmag.com/history/this-1600-year-old-goblet-shows-that-the-romans-were-nanotechnology-pioneers-787224/, accessed: 2019-09-06.

[67] K. G. Budden, *Radio waves in the ionosphere.* Cambridge University Press, 2009.

[68] R. Fitzpatrick, *Plasma physics: an introduction.* CRC Press, 2014.

[69] G. Strang, *Introduction to linear algebra.* Wellesley-Cambridge Press Wellesley, MA, 1993, vol. 3.

[70] K. C. Yeh and C.-H. Liu, "Radio wave scintillations in the ionosphere," *Proceedings of the IEEE*, vol. 70, no. 4, pp. 324–360, 1982.

[71] J. Kraus, *Electromagnetics.* McGraw-Hill, 1984.

[72] Wikipedia, "Circular polarization," https://en.wikipedia.org/wiki/Circular_polarization.

[73] Q. Zhan, "Cylindrical vector beams: from mathematical concepts to applications," *Advances in Optics and Photonics*, vol. 1, no. 1, pp. 1–57, 2009.

[74] H. Haus, *Electromagnetic Noise and Quantum Optical Measurements*, ser. Advanced Texts in Physics. Springer Berlin Heidelberg, 2000.

[75] W. C. Chew, "Lectures on theory of microwave and optical waveguides, for ECE 531 at UIUC," *https://engineering.purdue.edu/wcchew/course/tgwAll20160215.pdf*, 2016.

[76] L. Brillouin, *Wave propagation and group velocity.* Academic Press, 1960.

[77] R. Plonsey and R. E. Collin, *Principles and applications of electromagnetic fields.* McGraw-Hill, 1961.

[78] M. N. Sadiku, *Elements of electromagnetics.* Oxford University Press, 2014.

[79] A. Wadhwa, A. L. Dal, and N. Malhotra, "Transmission media," https://www.slideshare.net/abhishekwadhwa786/transmission-media-9416228.

[80] P. H. Smith, "Transmission line calculator," *Electronics*, vol. 12, no. 1, pp. 29–31, 1939.

[81] F. B. Hildebrand, *Advanced calculus for applications.* Prentice-Hall, 1962.

[82] J. Schutt-Aine, "Experiment02-coaxial transmission line measurement using slotted line," http://emlab.uiuc.edu/ece451/ECE451Lab02.pdf.

[83] D. M. Pozar, E. J. K. Knapp, and J. B. Mead, "ECE 584 microwave engineering laboratory notebook," http://www.ecs.umass.edu/ece/ece584/ECE584_lab_manual.pdf, 2004.

[84] R. E. Collin, *Field theory of guided waves.* McGraw-Hill, 1960.

[85] Q. S. Liu, S. Sun, and W. C. Chew, “A potential-based integral equation method for low-frequency electromagnetic problems,” *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 3, pp. 1413–1426, 2018.

[86] M. Born and E. Wolf, *Principles of optics: electromagnetic theory of propagation, interference and diffraction of light.* Pergamon, 1986, first edition 1959.

[87] Wikipedia, “Snell’s law,” https://en.wikipedia.org/wiki/Snell’s_law.

[88] G. Tyras, *Radiation and propagation of electromagnetic waves.* Academic Press, 1969.

[89] L. Brekhovskikh, *Waves in layered media.* Academic Press, 1980.

[90] Scholarpedia, “Goos-hanchen effect,” http://www.scholarpedia.org/article/Goos-Hanchen_effect.

[91] K. Kao and G. A. Hockham, “Dielectric-fibre surface waveguides for optical frequencies,” in *Proceedings of the Institution of Electrical Engineers*, vol. 113, no. 7. IET, 1966, pp. 1151–1158.

[92] E. Glytsis, “Slab waveguide fundamentals,” http://users.ntua.gr/eglytsis/IO/Slab_Waveguides_p.pdf, 2018.

[93] Wikipedia, “Optical fiber,” https://en.wikipedia.org/wiki/Optical_fiber.

[94] Atlantic Cable, “1869 indo-european cable,” https://atlantic-cable.com/Cables/1869IndoEur/index.htm.

[95] Wikipedia, “Submarine communications cable,” https://en.wikipedia.org/wiki/Submarine_communications_cable.

[96] D. Brewster, “On the laws which regulate the polarisation of light by reflexion from transparent bodies,” *Philosophical Transactions of the Royal Society of London*, vol. 105, pp. 125–159, 1815.

[97] Wikipedia, “Brewster’s angle,” https://en.wikipedia.org/wiki/Brewster’s_angle.

[98] H. Raether, “Surface plasmons on smooth surfaces,” in *Surface plasmons on smooth and rough surfaces and on gratings.* Springer, 1988, pp. 4–39.

[99] E. Kretschmann and H. Raether, “Radiative decay of non radiative surface plasmons excited by light,” *Zeitschrift für Naturforschung A*, vol. 23, no. 12, pp. 2135–2136, 1968.

[100] Wikipedia, “Surface plasmon,” https://en.wikipedia.org/wiki/Surface_plasmon.

[101] Wikimedia, “Gaussian wave packet,” https://commons.wikimedia.org/wiki/File:Gaussian_wave_packet.svg.

[102] Wikipedia, “Charles K. Kao,” https://en.wikipedia.org/wiki/Charles_K._Kao.

[103] H. B. Callen and T. A. Welton, “Irreversibility and generalized noise,” *Physical Review*, vol. 83, no. 1, p. 34, 1951.

[104] R. Kubo, "The fluctuation-dissipation theorem," *Reports on progress in physics*, vol. 29, no. 1, p. 255, 1966.

[105] C. Lee, S. Lee, and S. Chuang, "Plot of modal field distribution in rectangular and circular waveguides," *IEEE transactions on microwave theory and techniques*, vol. 33, no. 3, pp. 271–274, 1985.

[106] W. C. Chew, *Waves and Fields in Inhomogeneous Media*. IEEE Press, 1996.

[107] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions: with formulas, graphs, and mathematical tables*. Courier Corporation, 1965, vol. 55.

[108] "Handbook of mathematical functions: with formulas, graphs, and mathematical tables."